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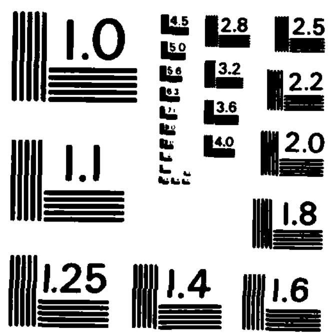

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BIFURCATION AND STABILITY OF COMPLEX FLOWS

FINAL REPORT

by

S. ROSENBLAT

OCTOBER 10, 1985

U. S. ARMY RESEARCH OFFICE

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19. KEY WORDS *(Continue on reverse side if necessary and identify by block number)*

Stability of fluid flows. Bifurcation phenomena. Viscoelastic fluids. Nonlinear hydrodynamic stability.

20. ABSTRACT *(Continue on reverse side if necessary and identify by block number)*

The work performed under this Contract was principally concerned with the theoretical investigation of bifurcation and stability phenomena in complex fluid flows. A major part of the effort was devoted to the study of viscoelastic fluids, with special emphasis on problems concerned with convective instabilities. Both model systems and fluid governing equations were investigated. The dependence of predicted behavior on constitutive relations became very clear. Other convection-type problems

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SECURITY CLASSIFICATION OF THIS PAGE *(When Data Entered)* considered included coupled thermal loops and convection patterns in the presence of a melting-freezing interface.

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1. TECHNICAL REPORT

1.1 INTRODUCTION

The aim of the work performed under this Contract was the theoretical investigation of bifurcation and stability phenomena in complex fluids (especially viscoelastic fluids) and in complex flow situations. A number of specific studies were performed covering a wide range of fluid flow phenomena, revealing in several instances unexpected and interesting behavior. The major thrust of the work, as it turned out, concerned problems in viscoelastic fluids, but other flow configurations involving Newtonian fluids were also studied. The unifying theme of the work was the application of bifurcation and stability theory to flows of non-standard type, with special emphasis on the influence of complex structure on classical results. The various problems that were investigated are outlined in the following Sections.

1.2 NONLINEAR STABILITY OF MODULATED CONVECTION

We have studied the problem of nonlinear instability of a layer of fluid heated from below in the case that the applied temperature at the lower surface varied sinusoidally in time about a non-zero mean. A linear analysis for this situation was performed several years ago, and indicated a change in the critical value of the Rayleigh number based on the mean temperature. Our primary interest in the nonlinear problem was to determine the direction of the bifurcating solution and the geometry of the convection pattern. The problem was solved using a perturbation procedure, and it was found that a subcritical bifurcation occurred (where a supercritical bifurcation occurs in the constant-gradient case), and that for a range of Rayleigh numbers the stable, preferred pattern comprises hexagonal cells rather than rolls.

1.3 NONLINEAR CONVECTION IN VISCOELASTIC FLUIDS

We have investigated the onset of convection in a layer of viscoelastic liquid heated from below. According to linear theory it is possible for a sufficiently elastic liquid to become destabilized by oscillatory disturbances, by contrast with a Newtonian or only slightly elastic liquid, which becomes unstable to steady disturbances. In the nonlinear theory we have considered a number of different constitutive relations, and have investigated the interaction between the inertial nonlinearity and the material nonlinearity. It was determined that these nonlinearities could be in competition, to the extent that the supercritical bifurcation of the Newtonian fluid could be replaced by a subcritical bifurcation under certain conditions. The detailed results depend on the particular constitutive equation, and so might be used as the basis for an experimental verification of a given constitutive relation.

1.4 VISCOELASTIC STABILITY MODELS

Through the medium of a model partial differential equation, we have studied the influence on bifurcation phenomena of viscoelastic properties in fluids. This model extends a classical model for bifurcation to convection to include a memory integral that represents fluid elasticity. The integral includes a parameter λ that measures elastic effects. When λ is less than some critical value only steady, supercritical bifurcation occurs, but when λ exceeds the critical value both steady and periodic bifurcation appear. A close analysis has been made when λ is in the neighborhood of the critical value, and some complex bifurcation phenomena have been revealed, such as the coexistence of a steady and a periodic branch, and the possibility of jumping between branches. Generally, however, it is found that periodic solutions tend to be only local in their appearance, and that the usual state is that of steady convection. The model gives insights into some fundamental consequences of elasticity in fluids.

1.5 VISCOELASTIC THERMOCAPILLARY INSTABILITY

We have treated the linear stability problem for a layer of viscoelastic liquid heated from below in the case that the instability is driven by surface tension gradients and measured by a Marangoni number. When the elasticity is small the conduction state becomes unstable at the same Marangoni number as for a Newtonian liquid, and is destabilized by a steady disturbance. When the elasticity is large, however, the instability can set in at a much lower Marangoni number and the critical disturbance is periodic. Detailed calculations have been performed for various values of one or more elasticity parameters. The results are similar to those found by other authors in the case of buoyancy-driven instability.

1.6 STABILITY OF HIGHLY ELASTIC LIQUIDS

It has been conjectured for some time that the appearance of fracture in polymer melts under shear may be due to an instability which occurs in the governing equations at very low Reynolds numbers but high values of an elasticity parameter. To test this hypothesis we performed some computations to determine the stability characteristics of plane shearing flows, using different types of constitutive relations to represent fluid elasticity. None of the calculations performed to date have revealed a low Reynolds number instability, but the whole range of parameters was not explored and the issue remains open.

1.7 COUPLED LORENZ OSCILLATORS

We have considered two thermal loops, each consisting of a solid tube filled with fluid bent into a closed loop, that are thermally, but not hydrodynamically, coupled. Temperature differences across the loops result in buoyancy driven fluid flows that convey heat across the system. When the tube radius is very small compared with the loop radius, approximate governing equations are obtained that resemble two coupled Lorenz systems. The problem has been analyzed, and it has been determined that when the temperature differences for the two loops are equal, there are co-current as well as counter-current flows. The latter convect less heat than the former and these can coexist at the same Rayleigh number as stable convective states. The chaotic states, it is found, can combine co-current and counter-current spatial orientations. When the imposed temperature differences are unequal, there is an imperfect bifurcation as well as flow reversal and a jump in heat transport.

1.8 CONVECTION WITH A MELTING - FREEZING INTERFACE

We considered the problem of convection which was strongly coupled with the corrugations of a melting-freezing interface. A layer of fluid is heated from below and the temperatures adjusted so that the upper part is frozen and there is a solid-liquid interface. The patterns of convective flow and interfacial corrugations were studied both experimentally and theoretically. In the experiments it was found that a large ratio of solid to liquid thickness resulted in hexagonal convection, while small ratios resulted in roll convection. A weakly nonlinear stability theory was developed which gave good qualitative agreement with the experimental observations. It was shown that the onset of hexagonal convection is accompanied by a jump in the mean position of the solid - liquid interface.

1.9 VISCOELASTIC EKMAN LAYER

Solutions were computed for the Ekman layer flow of a viscoelastic fluid for a range of constitutive relations. This problem is relevant to the behavior of liquid-filled projectiles. It was found that the nature and extent of shear thinning had a decisive effect on the flow field; in the absence of shear thinning the flow was identical with that in a Newtonian fluid, while in the presence of shear thinning the layer was thinner than in the Newtonian case. Some relativity common constitutive relations led to the nonexistence of a solution.

1.10 VISCOELASTIC RIVULET FLOW

We have computed the solution for the flow of a viscoelastic liquid in a straight rivulet. Various constitutive relations were applied and it was shown that the dominant viscoelastic characteristic was the second normal stress difference predicted by the constitutive relation. It was shown that in certain circumstances no solution could exist.

1.11 SHEAR FLOWS WITH TEMPERATURE - DEPENDENT VISCOSITY

Solutions were computed for the shear flow of a liquid with an Arrhenius - law dependence of viscosity on temperature, and with viscous dissipation taken into account. The presence of either one or three solutions was demonstrated, and the hysteresis effect associated with the latter case was revealed.

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3. PERSONNEL

The following personnel participated in and were partially supported under this contract.

S. Rosenblat, Professor, Illinois Institute of Technology (Principal Investigator).

S. H. Davis, Professor, Northwestern University (Principal Investigator).

M. Roppo, Research assistant, Northwestern University: Ph.D. awarded January 1985.

J. A. Horwitz, Research assistant, Illinois Institute of Technology: M.S. awarded December 1983.

D. Getachew, Research assistant, Illinois Institute of Technology: M.S. awarded December 1983.

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W. E. Olmstead, Professor, Northwestern University.

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